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Cosmology of Dilatonic Brane World

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Abstract

We study cosmological solutions in the dilatonic brane world models. The effective four-dimensional equations on the brane are analyzed for the models with one positive tension brane and two branes with tensions of opposite signs. Just as in the non-dilatonic brane case, the conventional Friedmann equations of the four-dimensional universe are reproduced to the leading order in matter energy density for the model with one brane and the introduction of a radion potential is required in order to reproduce the Friedmann equations with the correct sign for the model with two branes.

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Over the past couple of years, much attention has been paid to theories with extra dimensions [1, 2, 3, 4, 5, 6], as such theories open up the possibility of solving the hierarchy problem in particle physics and make the string scale and the extra dimensions accessible to the future accelerators. Such theories assume that fields of the Standard Model are confined to a three-brane embedded in higher-dimensional spacetime, whereas gravity can propagate in the bulk. In the scenario proposed by Randall and Sundrum (RS) [4, 5, 6], four-dimensional gravity with a negligible correction is reproduced even with an infinitely large extra dimension, since the gravity is effectively localized on the brane.

Lots of effort has been made to understand cosmology in brane models. Initially, it was observed [7] that brane models (with vanishing bulk cosmological constant Λ and brane tension σ) have non-conventional cosmological solutions where the Hubble parameter H is proportional to the energy density ϱ of matter on the brane, whereas in standard conventional cosmology $H \propto \varrho$. This problem was resolved [8, 9, 10, 11] by considering the RS brane world model, i.e. by setting Λ and σ to be nonzero. By assuming that $\varrho \ll \sigma$, one reproduces the conventional cosmology to the leading order in ϱ for the RS model with one brane and an infinitely large extra dimension (called the RS2 model). However, for the RS model with two branes (called the RS1 model), the cosmological solution on the visible brane (the TeV brane) has the wrong-signed Friedmann equations. Furthermore, the matter energy densities ϱ and ϱ_* on the invisible and the visible branes are correlated to have the opposite signs, which leads to the unphysical result that the matter energy density on either of the branes has to be negative. As was speculated in Refs. [12, 7, 8] and later explicitly shown in Ref. [13] (see also Ref. [14] for the brane world cosmology with a radion stabilizing potential), such undesirable results are resolved by including a radion stabilizing potential [15, 12, 16]. In the limit of very heavy radion mass, one of the equations of motion which correlated the energy densities on the two branes disappears and the standard cosmology is reproduced. Another approach to resolve the problem of the brane cosmology emphasized in Ref. [7] was proposed in Refs. [17, 18], where it is shown that the standard Friedmann equations can be reproduced (without any approximation) by introducing the nonzero extra-spatial component of the bulk energy-momentum tensor (even without the bulk cosmological constant and the brane tension). Such nonzero bulk energy-momentum tensor component acts to stabilize the radius of the extra spatial dimension even in the absence of the second brane.

It was found out [19, 20] that the RS type brane world scenario can be extended to the dilatonic branes, since gravity can also be localized on the dilatonic branes when the warp factor decreases, for which case the tension of the brane is positive. Dilatonic domain walls appear in string theories quite often when (intersecting) branes in ten or eleven dimensions are compactified. It is the purpose of this note to study cosmological solutions in the dilatonic brane world. (The previous works on the brane world cosmology with the bulk dilatonic or Brans-Dicke scalar can be found in Refs. [21, 22, 23, 24].)

We consider the following action:

$$S=\int d^5x\sqrt{-\hat{g}}\left[\frac{1}{2\kappa_5^2}\mathcal{R}-\frac{4}{3}\partial_M\phi\partial^M\phi-e^{-2\alpha\phi}\Lambda\right]+\int d^4x\sqrt{-g}\left[\mathcal{L}_{mat}-\sigma e^{-\alpha\phi_0}\right], \quad (1)$$

where σ is the tension of the brane located at $y=0$, $\phi_0\equiv\phi|_{y=0}$, and $\mathcal{L}_{mat}$ is the Lagrangian density for all the matter confined on the three-brane.

When $\mathcal{L}_{mat}=0$, the equations of motion following from the action (1) admit the following static three-brane solution [19]:

$$\begin{aligned} g_{MN}dx^Mdx^N &= \mathcal{W}\left[-dt^2+(dx^1)^2+(dx^2)^2+(dx^3)^2\right]+dy^2, \\ \phi &= \frac{1}{\alpha}\ln(1-K|y|)+\phi_0, \qquad \mathcal{W}=(1-K|y|)^{\frac{16\kappa_5^2}{9\alpha^2}}, \end{aligned} \quad (2)$$

where the coefficient K is determined by Λ in the following way:

$$K=\frac{3\alpha^2}{2}e^{-\alpha\phi_0}\sqrt{\frac{3\Lambda}{9\alpha^2-32\kappa_5^2}}, \quad (3)$$

and ϕ_0 is an arbitrary constant. The following fine-tuned value of σ in terms of Λ is fixed by the boundary condition at $y=0$:

$$\sigma=8\sqrt{\frac{3\Lambda}{9\alpha^2-32\kappa_5^2}}. \quad (4)$$

It was shown [19, 25, 20] that as long as σ takes positive value given by Eq. (4) (i.e. the case with the $\mathbf{Z}_2$-symmetry and naked singularities on both sides of the brane) there exists normalizable Kaluza-Klein zero mode for the bulk graviton. Thereby, the RS type brane world scenario can be extended to the dilatonic domain wall case.

When $\mathcal{L}_{mat}\neq 0$, even if σ takes the fine tuned value (4), the brane world becomes no longer static, i.e. the brane world undergoes cosmological expansion. We are interested in the cosmological model for which the principle of homogeneity and isotropy in the three-dimensional space of the brane universe is satisfied. On the other hand, the presence of the brane breaks the isometry along the extra spatial direction. The general form of the metric Ansatz satisfying these requirements is

$$\hat{g}_{MN}dx^Mdx^N=-n^2(t,y)dt^2+a^2(t,y)\gamma_{ij}dx^idx^j+b^2(t,y)dy^2. \quad (5)$$

Here, γ_{ij} is the maximally symmetric three-dimensional metric given in the Cartesian and spherical coordinates by

$$\gamma_{ij}dx^idx^j=\left(1+\frac{k}{4}\delta_{mn}x^mx^n\right)^{-2}\delta_{ij}dx^idx^j=\frac{dr^2}{1-kr^2}+r^2(d\theta^2+\sin^2\theta d\phi^2), \quad (6)$$

where $k=-1,0,1$ respectively for the three-dimensional space with the negative, zero and positive spatial curvature.

By varying the action (1) with respect to $\hat{g}_{MN}$, one obtains the Einstein's equations

$$\mathcal{G}_{MN}=\kappa_5^2 T_{MN}, \tag{7}$$

with the energy-momentum tensor given by

$$\begin{aligned} T_{MN} &= -\frac{4}{3}\hat{g}_{MN}\partial_P\phi\partial^P\phi+\frac{8}{3}\partial_M\phi\partial_N\phi-\hat{g}_{MN}e^{-2\alpha\phi}\Lambda \\ &\quad -\frac{\sigma}{b}e^{-\alpha\phi}\delta(y)\delta^\mu_M\delta^\nu_N g_{\mu\nu}+\frac{1}{b}\delta(y)\delta^\mu_M\delta^\nu_N T^{mat}_{\mu\nu}, \end{aligned} \tag{8}$$

where $T^{mat}_{\mu\nu}=-\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_{mat})}{\delta g^{\mu\nu}}$ is the energy-momentum tensor of the matter fields on the three-brane. Since we wish to model the matter in the brane universe by a perfect fluid, $T^{mat}_{\mu\nu}$ in the comoving coordinates takes the following form:

$$T^{mat\,\mu}{}_{\nu}=\mathrm{diag}(-\varrho,\wp,\wp,\wp), \tag{9}$$

where ϱ and $\wp$ are the energy density and pressure of matter on the three-brane as measure in the rest frame. So, the Einstein's equations (7) take the following form:

$$\begin{aligned} &\frac{3}{n^2}\frac{\dot a}{a}\left(\frac{\dot a}{a}+\frac{\dot b}{b}\right)-\frac{3}{b^2}\left[\frac{a'}{a}\left(\frac{a'}{a}-\frac{b'}{b}\right)+\frac{a''}{a}\right]+\frac{3k}{a^2}= \\ &\qquad \kappa_5^2\left[\frac{4}{3}n^{-2}\dot\phi^2+\frac{4}{3}b^{-2}\phi'^2+e^{-2\alpha\phi}\Lambda+(\sigma e^{-\alpha\phi}+\varrho)\frac{\delta(y)}{b}\right], \end{aligned} \tag{10}$$

$$\begin{aligned} &\frac{1}{b^2}\left[\frac{a'}{a}\left(2\frac{n'}{n}+\frac{a'}{a}\right)-\frac{b'}{b}\left(\frac{n'}{n}+2\frac{a'}{a}\right)+2\frac{a''}{a}+\frac{n''}{n}\right] \\ &\qquad +\frac{1}{n^2}\left[\frac{\dot a}{a}\left(2\frac{\dot n}{n}-\frac{\dot a}{a}\right)+\frac{\dot b}{b}\left(\frac{\dot n}{n}-2\frac{\dot a}{a}\right)-2\frac{\ddot a}{a}-\frac{\ddot b}{b}\right]-\frac{k}{a^2}= \\ &\qquad \kappa_5^2\left[\frac{4}{3}n^{-2}\dot\phi^2-\frac{4}{3}b^{-2}\phi'^2-e^{-2\alpha\phi}\Lambda+(\wp-\sigma e^{-\alpha\phi})\frac{\delta(y)}{b}\right], \end{aligned} \tag{11}$$

$$\frac{n'}{n}\frac{\dot a}{a}+\frac{a'}{a}\frac{\dot b}{b}-\frac{\dot a'}{a}=\frac{8}{9}\kappa_5^2\dot\phi\phi', \tag{12}$$

$$\frac{3}{b^2}\frac{a'}{a}\left(\frac{a'}{a}+\frac{n'}{n}\right)-\frac{3}{n^2}\left[\frac{\dot a}{a}\left(\frac{\dot a}{a}-\frac{\dot n}{n}\right)+\frac{\ddot a}{a}\right]-\frac{3k}{a^2}=\kappa_5^2\left[\frac{4}{3}n^{-2}\dot\phi^2+\frac{4}{3}b^{-2}\phi'^2-e^{-2\alpha\phi}\Lambda\right], \tag{13}$$

where the overdot and the prime respectively denote derivatives w.r.t. t and y.

With the assumption of homogeneity and isotropy on the brane world, the dilaton ϕ does not depend on the spatial coordinates x^i $(i = 1, 2, 3)$ of the three-brane. So, the equation of motion for the dilaton takes the following form:

$$\frac{8}{3}\frac{1}{n^2}\left[\ddot{\phi} - \dot{\phi}\left(\frac{\dot{n}}{n} - 3\frac{\dot{a}}{a} - \frac{\dot{b}}{b}\right)\right] - \frac{8}{3}\frac{1}{b^2}\left[\phi'' + \phi'\left(\frac{n'}{n} + 3\frac{a'}{a} - \frac{b'}{b}\right)\right]$$
$$= 2\alpha\Lambda e^{-2\alpha\phi} + \alpha\sigma e^{-\alpha\phi}\frac{\delta(y)}{b}. \qquad (14)$$

We assume that a solution to the above equations of motion is continuous everywhere, especially across $y = 0$, where the brane is located, in order to have a well-defined geometry. However, its derivatives w.r.t. y are discontinuous at $y = 0$ due to the δ-function like brane source there. The following boundary conditions on the first derivatives of the metric components at $y = 0$ are obtained by integrating Eqs. (10) and (11) over the infinitesimal interval around $y = 0$ w.r.t. y:

$$\frac{[a']_0}{a_0 b_0} = -\frac{\kappa_5^2}{3}(\sigma e^{-\alpha\phi_0} + \varrho), \qquad (15)$$

$$\frac{[n']_0}{n_0 b_0} = -\frac{\kappa_5^2}{3}(\sigma e^{-\alpha\phi_0} - 3\wp - 2\varrho), \qquad (16)$$

where the subscript 0 denotes quantities evaluated at $y = 0$, e.g. $a_0(t) \equiv a(t, 0)$, and $[F]_0 \equiv F(0^+) - F(0^-)$ denotes the jump of $F(y)$ across $y = 0$. Similarly, from the dilaton equation (14), one obtains the following boundary condition on the first derivative of ϕ at $y = 0$:

$$\frac{[\phi']_0}{b_0} = -\frac{3}{8}\alpha\sigma e^{-\alpha\phi_0}. \qquad (17)$$

The effective four-dimensional equations of motion on the three-brane can be obtained [7] by taking the jumps and the mean values of the above five-dimensional equations of motion across $y = 0$ and then applying the boundary conditions (15-17) on the first derivatives. Here, the mean value of a function F across $y = 0$ is defined as $\sharp F\sharp \equiv [F(0^+) + F(0^-)]/2$. In this paper, we consider solutions invariant under the $\mathbf{Z}_2$ symmetry, $y \to -y$, i.e. the solutions depending on y through $|y|$. Then, the mean values of the first derivatives across $y = 0$ vanish. We also note that it is always possible to choose a gauge so that $n_0(t) \equiv n(t, 0)$ is constant without introducing the cross term $\hat{g}_{04}$. Making use of this fact, we scale the time coordinate t to be the cosmic time for the brane universe, namely $n_0 = 1$.

First, by taking the jump of the $(0, 4)$-component Einstein equation (12), one obtains the following conservation of energy equation for the brane universe with the scale factor a_0:

$$\dot{\varrho} + 3(\wp + \varrho)\frac{\dot{a}_0}{a_0} = 0. \qquad (18)$$

So, despite the energy flow along the y-direction, as indicated in Eq. (12), the energy conservation law in the brane universe takes the conventional form of the standard four-dimensional universe. A consequence for this fact is that for the brane matter satisfying the equation of state of the form $\wp = w\varrho$ with a constant w, the dependence of ϱ on a_0 has the usual form $\varrho \propto a_0^{-3(1+w)}$.

Next, by taking the mean value of the $(4,4)$-component Einstein equation (13) across $y = 0$, one obtains the following Friedmann-type equation of the brane universe:

$$\begin{aligned}\frac{\dot{a}_0^2}{a_0^2} + \frac{\ddot{a}_0}{a_0} + \frac{k}{a_0^2} &= \frac{\kappa_5^4}{36}(\varrho - 3\wp)\sigma e^{-\alpha\phi_0} + \left(\frac{\kappa_5^4}{18}\sigma^2 - \frac{\kappa_5^2}{64}\alpha^2\sigma^2 + \frac{\kappa_5^2}{3}\Lambda\right) e^{-2\alpha\phi_0} \\ &\quad -\frac{\kappa_5^4}{36}\varrho(\varrho + 3\wp) - \frac{4}{9}\kappa_5^2\dot{\phi}_0^2. \end{aligned} \tag{19}$$

The $\sim e^{-2\alpha\phi_0}$ term on the RHS vanishes when σ takes the fine-tuned value (4). When σ is bigger [smaller] than the fine-tuned value, the effective cosmological constant term behaving as $\sim e^{-2\alpha\phi_0}$ is positive [negative]. However, unlike the non-dilatonic brane world case, we have additional negative contribution (the last term on the RHS) to the cosmological constant from the varying dilaton field with t. This can be understood from Eq. (12), which indicates the flow of the dilaton energy along the y-direction. The extremely small positive cosmological constant observed in our universe restricts the dilaton to vary very slowly on the brane or to be stabilized due to some mechanism. When σ does not take the fine-tuned value, the varying ϕ_0 implies the varying cosmological constant in the brane universe with t. If we assume $\varrho \ll \sigma$, then to the leading order in ϱ we have $H^2 \propto \varrho$ as in conventional cosmology. However, unlike the non-dilatonic brane world case, the coefficient of the $\sim \varrho$ term varies with t, if $\dot{\phi}_0 \neq 0$. The $\Lambda = 0$ case corresponds to cosmology in the self-tuning brane world [26, 27]. In this case, the four-dimensional effective cosmological constant term ($\sim e^{-2\alpha\phi_0}$) is nonzero for a general value of α, as was previously observed [25, 28] in the four-dimensional effective action. This cosmological constant term vanishes when $\alpha^2 = 32\kappa_5^2/9$. Contrary to the result in Ref. [28], this nonzero cosmological constant cannot be cancelled by introducing another brane with the fine-tuned tension, since the RHS of Eq. (19) generally does not receive contribution from another brane at different y. This difference may be attributed to the fact that the Friedmann-like equation (19) contains information local in the y-direction (i.e. only from $y = 0$), whereas the four-dimensional effective action contains contribution from all the possible values of y. Note, so far, we have not assumed the radius b of the extra dimension to be constant.

We now discuss an approximate solution to the equations of motion. We assume the radius of the extra space to be stable, i.e. $b = b_0 = const$, even in the absence of a stabilizing potential. We assume that the brane matter satisfies the equation of state of the form $\wp = w\varrho$ with a constant w. The generalization of the static brane solution

(2) can be parameterized as

$$
\begin{aligned}
a(t,y) &= a_0(t)\left[1+A(t)|y|+A_2(t)y^2+\cdots\right]^{\frac{8\kappa_5^2}{9\alpha^2}}, \\
n(t,y) &= \left[1+N(t)|y|+N_2(t)y^2+\cdots\right]^{\frac{8\kappa_5^2}{9\alpha^2}}, \\
\phi(t,y) &= \frac{1}{\alpha}\ln\left[1+\Phi(t)|y|+\Phi_2(t)y^2+\cdots\right]+\phi_0(t).
\end{aligned} \tag{20}
$$

The coefficients of the $|y|$ terms can be determined by applying the boundary conditions (15-17) on the first derivatives. The resulting expressions are

$$
A=-Kb_0-\frac{3\alpha^2}{16}\varrho b_0, \quad N=-Kb_0+\frac{3\alpha^2}{16}(3w+2)\varrho b_0, \quad \Phi=-Kb_0. \tag{21}
$$

Note, in the presence of matter fields on the brane, $K=\frac{3\alpha^2}{16}e^{-\alpha\phi_0}\sigma$ changes with time, since $\phi_0=\phi(t,0)$ is in general a function of t. So, the time dependence of the above coefficients comes not only from $\varrho(t)$ but also from K. Of course, when there are no matter fields and σ takes the fine-tuned value (4), K is a constant and the above inflationary brane solution reduces to the static brane solution (2). The linear order (in $|y|$) part of the above non-static brane solution (20) suggests the following approximate solution, valid for any y, to the first order in ϱ:

$$
\begin{aligned}
a(t,y) &\approx a_0(t)(1-Kb_0|y|)^{\frac{8\kappa_5^2}{9\alpha^2}}[1+\varrho(t)f(t,y)], \\
n(t,y) &\approx (1-Kb_0|y|)^{\frac{8\kappa_5^2}{9\alpha^2}}[1+\varrho(t)g(t,y)], \\
\phi(t,y) &\approx \frac{1}{\alpha}\ln(1-Kb_0|y|)+\phi_0(t),
\end{aligned} \tag{22}
$$

where

$$
\begin{aligned}
f(t,y) &= -\frac{3\alpha^2}{32K}\left[(1-Kb_0|y|)^{-\frac{16\kappa_5^2}{9\alpha^2}}-1\right], \\
g(t,y) &= \frac{3\alpha^2(3w+2)}{32K}\left[(1-Kb_0|y|)^{-\frac{16\kappa_5^2}{9\alpha^2}}-1\right].
\end{aligned} \tag{23}
$$

This approximate solution reduces to the one obtained in Ref. [13] in the non-dilatonic limit $\alpha\to 0$.

Just like the RS1 model [4], one can introduce another brane at some fixed distance from the original brane. We put this second brane at $y=1/2$. We denote the energy density and pressure of the matter fields on the brane at $y=1/2$ as ϱ_* and $\wp_*$. The tension of the brane at $y=1/2$ is denoted as σ_*. Then, the equations of motion (10-14) have additional δ-function terms $\sim\delta(y-1/2)$ associated with the additional brane and brane matter at $y=1/2$. The metric components and the dilaton satisfy additional boundary conditions of the form (15-17) at $y = 1/2$ but with the respective quantities corresponding to those at $y = 1/2$. Just as in the non-dilatonic brane case in the previous works [8, 9] and as was pointed out [7] to be a generic topological constraint for a model with compact extra dimension and two branes, the requirement of the stable radius (i.e. $b_0 = const$) without a stabilizing potential restricts ϱ and ϱ_* to be related to one another in the following way:

$$\varrho_* = -\mathcal{W}_{1/2}\varrho, \tag{24}$$

where $\mathcal{W}_{1/2} \equiv (1-Kb_0/2)^{\frac{16\kappa_5^2}{9\alpha^2}}$ with time-dependent $K = \frac{3\alpha^2}{16}e^{-\alpha\phi_0(t)}\sigma$ (thereby, the ratio ϱ_*/ϱ in general changes with time). [This constraint is understood [13] as a fine-tuning of the matter energy densities on the two branes required to maintain the constant radius b of the extra dimension even in the absence of the stabilizing potential.] As a consequence, we have unphysical result that the matter energy density on either of the branes has to be negative. Since $\sigma_* < 0$, the Friedmann equations on the second brane has the opposite sign from those of the conventional cosmology. These problems can be resolved by introducing a radion potential $U(b)$, which stabilizes the radius b of the extra dimension, as was explicitly shown in Ref. [13]. The energy momentum tensor (8) receives the following additional contribution from the radion potential:

$$T^{rad}_{00} = -n^2U(b), \qquad T^{rad}_{ij} = a^2U(b)\gamma_{ij}, \qquad T^{rad}_{44} = b^2\left[U(b) + bU'(b)\right]. \tag{25}$$

Assuming that the radion mass m_{rad} is very heavy and the radion potential is approximated to $U(b) \approx M_b^5\left(\frac{b-b_0}{b_0}\right)^2$ (with $M_b^5 \propto m_{rad}$ assumed to be the largest mass scale of the theory) near its minimum, one has $b = b_0$ from the (4,4)-component Einstein's equation without constraining the matter energy densities on the two branes. Then, the remaining components of the Einstein's equations averaged over the bulk (by integrating w.r.t. y) lead to the conventional Friedmann equations. Note, however that since we averaged the Einstein's equations over y, the effective Friedmann equations contain contributions from matter fields on both branes, unlike the cosmological solutions of other related works and the solutions we discussed in the above, for which the effective Friedmann equations are local in y.

Finally, we comment on the case in which the matter fields on the three-brane couple to the dilaton field, i.e. the $\delta\mathcal{L}_{mat}/\delta\phi \neq 0$ case. If we assume that there exists the frame in which the matter fields decouple from the dilaton field (just like the Jordan frame of the Brans-Dicke theory), namely the matter fields on the three-brane are minimally coupled with respect to a Weyl rescaled metric $\tilde{g}_{\mu\nu} = \Omega^2(\phi)g_{\mu\nu}$, then the dilaton equation (14) is modified to

$$\frac{8}{3}\frac{1}{n^2}\left[\ddot{\phi} - \dot{\phi}\left(\frac{\dot{n}}{n} - 3\frac{\dot{a}}{a} - \frac{\dot{b}}{b}\right)\right] - \frac{8}{3}\frac{1}{b^2}\left[\phi'' + \phi'\left(\frac{n'}{n} + 3\frac{a'}{a} - \frac{b'}{b}\right)\right]$$

$$= 2\alpha\Lambda e^{-2\alpha\phi} + \alpha\sigma e^{-\alpha\phi}\frac{\delta(y)}{b} + \frac{1}{\Omega}\frac{d\Omega}{d\phi}T^{mat\,\mu}{}_{\mu}\frac{\delta(y)}{b}. \tag{26}$$

So, the boundary condition (17) on the first derivative of ϕ at $y = 0$ is modified to

$$\frac{[\phi']_0}{b_0} = -\frac{3}{8}\alpha\sigma e^{-\alpha\phi_0} - \frac{3}{8}(3\wp - \varrho)\frac{1}{\Omega_0}\frac{d\Omega_0}{d\phi_0}, \tag{27}$$

where $\Omega_0 \equiv \Omega(\phi_0)$. Note, the second term on the RHS vanishes for the radiation dominated universe, for which $\wp = \varrho/3$. So, Eq. (19) is modified to

$$\frac{\dot{a}_0^2}{a_0^2} + \frac{\ddot{a}_0}{a_0} + \frac{k}{a_0^2} = \left(\frac{\kappa_5^4}{36} - \frac{\kappa_5^2}{32}\alpha\frac{1}{\Omega_0}\frac{d\Omega_0}{d\phi_0}\right)(\varrho - 3\wp)\sigma e^{-\alpha\phi_0} + \left(\frac{\kappa_5^4}{18}\sigma^2 - \frac{\kappa_5^2}{64}\alpha^2\sigma^2\right.$$
$$\left. + \frac{\kappa_5^2}{3}\Lambda\right)e^{-2\alpha\phi_0} - \frac{\kappa_5^4}{36}\varrho(\varrho + 3\wp) - \frac{\kappa_5^2}{64}(3\wp - \varrho)^2\frac{1}{\Omega_0^2}\left(\frac{d\Omega_0}{d\phi_0}\right)^2 - \frac{4}{9}\kappa_5^2\dot{\phi}_0^2. \tag{28}$$

In the limit $\varrho \ll \sigma$, the conventional Friedmann equations with $H^2 \sim \varrho$ behavior are reproduced to the leading order, however the coefficient is modified due to the coupling of the matter fields to ϕ. On the positive tension brane, in order for the Friedmann equations with the correct sign to be reproduced, α, ϕ_0 and Ω_0 are constrained to satisfy $\alpha\Omega_0^{-1}d\Omega_0/d\phi_0 < 8\kappa_5^2/9$. Particularly when $\Omega = e^{\beta\phi}$ with a constant β, this constraint restricts the allowed values of α and β to satisfy $\alpha\beta < 8\kappa_5^2/9$. The coupling of the matter fields to ϕ also induces another subleading correction (the second to the last term on the RHS) to the Friedmann equations.

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